

Evaluation of Optimization Methods with Multimodal Test Functions: A Comparative Approach

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General Terms

Swarm intelligence, nature-inspired algorithm.

Abstract

This paper attempts to put up a general review of common knowledge of dis-similar types of the intelligent optimization process, controlling techniques and algorithms like that swarm optimization or PSO. In this algorithm, we are deploying two models called as global – best and local – best, and hence we consider these models as in multimodal test functions. At last, the studies and results show that the methods are very highly competitive and also can be used as a suitable another approach to solving the problem of multi-object optimization when dealing with multimodal functions. We study different uncertainties from present approaches to addressing them and relationship of different processes is discussed. At last, we would like to propose some promising points are suggested for future research purpose.

Introduction

In the past some years ago, the optimization was only a curiosity. In 1995, optimization was first put forward by two scientists Dr. Eberhart and Dr. Kennedy in the domain of computing and artificial intelligence [1]. Nowadays PSO algorithms have so many significant attention by all researchers. Now swarm optimizations have more applications for researchers around the globe. Swarm optimization or PSO is the study of social behavior and cognitive knowledge [2]. It is a concept, which consists of accelerating the particle with respect to time. PSO has been a win-win position which is successfully implemented in almost all research areas. It is a population-based continuous non-linear optimization technic [3]. It is a logical step by step process which is a family of nature inspired. The most common algorithms are particle swarm optimization, ant colony optimization, and many more. It has several attributes. These algorithms are based on collective artificial intelligence [4]. Swarm is a collection of something that moves somewhere in large numbers like in the form of birds flock. It is a natural phenomenon as inspiration like a group of animals or flock of bird's sweeps across in the sky. Swarm intelligences (SIs) [13] are a relatively new approach to problem-solving. Swarm intelligences (SIs) are an artificial intelligence with self – governed the system, usually “multi-agent systems.” It simulates collectively social animal's behavior such as birds-flocking, fish schooling, ants, bees species, etc. In particle swarm optimization, each individual solution of problem is called the ‘particle’. Evolutionary algorithms (EAs) are the heuristic search method [5]. The social behavior of these species is presided by the art of learning, habits of adaptation and evolution. Examples: the ants easily search the shortest path to the food (source), and another one is the birds search their destination when they move from one place to another. The basic evolutionary technic proposed in the computing and artificial intelligence was genetic algorithms (GAs) [6]. These algorithms were introduced starting point on ‘survival of the fittest’ (Principle of Darwinian) [7]. The GAs algorithms have been used in the most applications of engineering and science [8]. Recent developments in GAs and various improvements in EAs, mimetic algorithms (MAs) [9], PSO and SFL [11] which inspired by the different-nature-based process.

The concept of particle swarm started as a simulation of a simple behavior of social species like bird flocking and other species. Genetic algorithm (GA) [6] is very similar to a PSO. PSO system is initialized with a population of the random solution. Each potential solution also assigned a randomized velocity in particle swarm optimization, and the potential solutions of the problem called ‘particles’. Every particle cause to continue a prior path of its co-ordinates in problem space search which is consisted with a best solution of the problem. This location is called pbest.

In ant colony optimization, whenever ants find food, it marks it return path or journey with a chemical substance called pheromones [12]. The shortest path will be reinforced by the pheromones further. Finally, the ants arrive at the shortest path.



Fig 1: Ant Colony Optimization

Ant characteristics: ant leaves pheromone when they travel for in search of food. Other ants follow the trail of pheromone if there is one side is more pheromone. Ants can figure out the shortest path.

Bee Algorithm: It has two steps:-

- ManageBeeActivity()
- Calculate Vectors()

Related Work

We see that Kennedy and Eberhart [14] proposed advice called PSO. It is a biological inspiration for the behavior of bird flocking. In this advice, we have learned as a distributed animal behavioral algorithm which may be performed in a broad or multi-dimensional space search. In this particle swarm optimization algorithm, the behavior of every andeach particle is influenced by either in the best global particle or may be the best local particle. In a PSO, we see by which particle swarm optimization allows particles to grant from their lapse of time i.e. past experiences. An interesting aspect is that there are many facts to extend particle swarm optimization to manage so many multi-objects. So, we will read here some multi-objects.

Parsopoulos and Vrahtis algorithm [15]: In this algorithm, it approaches an aggregating function out of all three approaches; first is the bang-bang aggregating weighted function, second is a conventional linear aggregation approach, and third is a dynamic aggregation approach.

Moore and Chapman algorithm [16]: Basically this algorithm is the key focus on Pareto dominance. In this algorithm, the best of performing a particle and a global search.

Hu and Eberhart algorithm [17]: This algorithm is based on dynamic neighborhood PSO. In this algorithm, lexicographic ordering is usedwith respect to time.

The swarm metaphor by Ray and Liew [18]: In this approach, it approaches Pareto dominance using particle swarm optimization.

Sir Deelman et al. [19] have done a remarkable scientific workflow on cloud mapping, planning, information-reuse in the field of job scheduling algorithm, etc. The mapping of job scheduling is an NP-complete problem which generally has two easy steps: First is job-scheduling with equally weighted up to two processors and another is scheduling jobs with continuous weights are more than two processors [20].

Lei et al. [21] stateby which the PSO algorithmsare to use best job schedule other than a genetic algorithm (GA) [6] on the basis of their experiment simulation for distributed computing.

Mostaghim and Teich [22] give a concept ofthe sigma approach that is a local – best solutions or guides for each particle. It refines convergence as well as diversity of a PSO which is nearly new for multiple objective optimizations. A concept of the Sigma method is familiar with compromise programming [23].

Hence we can say that past works have proposed so many heuristics approaches to particle swarm optimization and ant colony optimization. Previous tasks on cloud resources based on the high processing capabilities.

Problem declaration

Multiple criteria optimization can be proposed as:-
 Suppose that a variable vector satisfies some restrictions and also optimizes the vector function $F(v)$.
 Here we can propose it in the following way: -
 Vector $V = [V_1, V_2, \dots, V_N]$, if m =inequality constraints.

If we add the weight of inertia (w) in PSO, weight of inertia (w) is reduced to manage the impact of the limiting speeds on the present speed. Hence, impact of the compensation in the local and global exploration capabilities of a 'flight points' [24].

Some years ago, the meta-model was assisted by some approximate matching functions [25]. Motivations: The use of the approximate physical fitness function of the meta-model is motivated by the following reasons:-

- Each individual assessment of aptitude requires a lot of time. It is necessary to perform computational fluid dynamics (CFD) simulations [26] to evaluate the quality of a structure in dynamic design optimization. A CFD simulation is very expensive and also takes more than 10 hours for a single calculation in a high-end computer system [27].
- In an art design and music composition cases, the quality of the solutions must be evaluated by a human user [30]. The interactive EA framework can be adopted [28]. In the context of interactive multiple object EAs, meta-model approximation adequacy models [29] have also been used. In a real-time EO of control systems, simulations can be used for fitness evolutions. The estimation of physical fitness is an inherent problem if a person writes code for a part of the solutions and, therefore, the quality of the user cannot be evaluated [31].
- Extra aptitude assessments are also necessary when trying to find solid solutions [32].
- The main assumption in a global meta - model can be made to smooth the local optimum without changing the location of the global optimum [33].

Approximation methods: several methods can be adopted for the approximation of the aptitude in the evolutionary computation.

- Approximation of problems: try to use the original statement of the problem, which is the same as in the main problem to solve it easily.
 - There are many ad hoc methods used to solve some optimization problems. Example, random sampling is used to solve the problem instead of the complete sampling of an image using GAs [6].
 - Assignment of physical aptitude, imitation of physical fitness and physical condition: this approach is specific to EA. The goal is to develop the fitness of a user of other similar individuals is called inheritance [34]. An approach similar to the inheritance of fitness has also been suggested. Individuals are grouped that will be evaluated by the meta-model's fitness function. The idea of the imitation of fitness has been more sophisticated estimation methods [35]. Individuals in one of the two populations code or are part of the problem in two-level co-evolutionary optimization algorithm. The process is known as physical activity allocation to estimate physical fitness values to solve this problem [36]. The idea of imitation of physical conditioning has been broadened, and more sophisticated estimation methods have been developed [37].
- Mechanisms for meta-model incorporation: meta-models can take advantage of each EA element, including initialization, recombination, mutation, and physical fitness assessments.
- Use of approximation aptitude models to initialize the population.
 - Use of approximate physical fitness models through physical condition evaluations.

Population

It is a collection of N number of individuals. It is a specific factor of a population, if in the first generation of its evolution; it is its generic diversity. If and only if the population is very less, the scarcity of generic diversity can result in a population-based dominated by almost equivalent to chromosomes and then, after decoding the genes and evaluating function. On the other side, on too-large sides of the population, then mating individuals around different groups can produce newborn children who lack the good generic part of both or either of the parents.

Non - Pareto approach population

In VEGA approach, the sub-population is chosen from modified generation from every of its goal. To generate the new population from crossover and mutation method by using shuffling all subpopulation accordingly [38]. The key point of VEGAs was applied and used to fit in the particle swarm optimization framework and hence developing the algorithm. We are using two swarm particles to solve it at least in five benchmark problems like from f_1 to f_5 .

Every swarm particle was created to one of its goals accordingly. Especially the second swarm's best particle was used for the creating new velocities from the particles of the one swarm and also vice-versa. In another way,

second swarm's best positions could be used in an associationship with the second swarm's best particle for creation of the velocities of the one swarm and also the vice-versa.

Engineering problems on optimization have diluted to the different approaches of alternative solutions. Linear and non-linear programming processes fail due to reaching local optimum. Hence researchers have introduced evolutionary based algorithms to overcome these problems.

Ant colony optimization algorithm

Ant colony optimization evolves their nature-based social behavior. It is developed by Scientist-cum-Doctor Dorigo et al. [39]. Ants are compatible to search the nearest track from the nest to source of food. Ants deposited the pheromone trails, whenever ants travel. This is done in the form of communications.

When the ants search a source of food, they take the same food and when returning back to the home with food. In the ACO, ACO requires a variable S for each ant. Where t_{mn} = Ants associated pheromone concentrations; $m = 1, 2, \dots, S$ and $n = 1, 2, \dots, S$, when the process is start m random ants, an ant $k = 1, 2, \dots, m$ which is the solution string.

$t_{ij}(T) = pt_{ij}(T-1) + dt_{ij}$; where $T = 1, 2, 3, \dots, n$;

$t_{ij}(T)$ is the revised pheromone concentration at iteration T and $t_{ij}(T-1)$ at previous iteration, the concentration of pheromone, dt_{ij} = pheromone concentration change and p = rate of pheromone evaporation (value should be 0 to 1).

Particle swarm optimization

Various methods may be used to group particles into competitive and semi-independent flocks. The particles may associate to only one global flock of bird species. The simple concepts have been increasing effectively in a wide variety of problems. In 1995, PSO was firstly developed by Dr. J. Kennedy and Dr. R. Eberhart from inspiration of the studies of the behavior of waterfowl by biologist Frank Heppner [2]. It belongs to problem-solving techniques inspiration by evolution, such as evolution and genetic algorithms [6].

PSO algorithm

This algorithm has consisted of 'n' number of particles. The particles are changing its own conditions in the following three principles accordingly.

1. To change the most optimist position of the swarm
2. To keep its inertia
3. To change its most optimist position.

PSO is the nature-based social behavior of migrating birds flocking. These birds are

Each solution is a 'bird' when they are flocking to reach the destination, and each 'bird' is referring as a particle. We know that the particle swarm optimization simulates the social behavior of the birds flocking. Let us assume that to check the following scenario: Birds flocking randomly searches for a food source in a particular way [3]. If there are only one source of food in a particular space or field. Moreover, each and every bird do not know, that where are found the food or lying on the ground or somewhere else. Then, the best strategy is to search the source of food; then the most impact is that to follow the species that is closest to the source of food.

In particle swarm optimization, we know that we have learned from the current state. Hence we provided it to solve easily optimization problems. Each and every individual solution is a "species" in the optimization search space in the PSO. Hence we call it "particle."

In PSO, after finding the two best levels that are local best and global best, particle continuously updates their positions and velocities are the following from these equations.

$$Vel^{k+1} = Vel^k + K_1 * r^k * (lpbest^k - x_i) + K_2 * r^k * (glbest^k - x_i) \quad (1)$$

$$x_{i+1} = x_i + Vel^k \quad (2)$$

Vel^k is velocities of particles; x_i is the present solution of current state particle. Here $lpbest^k$ and $glbest^k$ are defined as indicated above. Random r^k is a random number and range should be between (from 0 to 1). K_1, K_2 is

learning factors. Usually $K_1 = K_2 = 3$.

Let a workflow programme say $W = (T, E)$ is analysis as a graph, in which $T = \{t_1, t_2, \dots, t_{n}\}$ have been as a set of a number of tasks and E may have been as a set of n number of edges. Where an edge $e_{i,j}$ may have been task form (t_i, t_j) , in which, where are $t_i =$ parent task and $t_j =$ child task. Hence we say that if all parent tasks are completed, then no one child tasks are executed. In fig. 1, every workflow application has a deadline d_w which is equal to the execution of the workflow application time limit. As we know cloud provider - Infrastructure as a Service (IaaS) offering a long range of virtual machines. The type of VM_i may be in the form of PVM_i which is equal to processing capacity, and another factor is CVM_i equal to cost per unit of time.

In our case, let VMs have some memory to run the workflow applications.

The performance variation is carried out by taking the suitable value of the processing capacities. In the form of floating point operations per second i.e. (FLOPS) and hence calculating a performance based degradation percentage deg_{VM_i} .

Unit of time taken t is generally provided by the service provider, belongs to the pay-per-use basis, and partial use of the VMs is calculated. We can understand it by a suitable example. For a while, $t = 60$ minutes and supposed to a VMs are used $t = 61$ minutes. Hence user has to promote for two cycle periods of each 60 minutes, which will be 120 minutes. The velocities of the particles in each dimension are set to V_{max} at maximum velocity.

PSO and Genetic algorithms comparison

- A. An initial population's random generation
- B. Recognition of fitness level for each object. It will depend proportional on the distance to the optimum.
- C. Population-based reproduction on fitness levels.
- D. If the conditions of requirements are met, then end. Else, come again to B point.

All above these are the most evolutionary techniques have described here. In comparison with the genetic algorithms (GA) [6], the controlling of knowledge exchange in particle swarm optimization are significantly different. In genetic algorithm, the chromosomes shares the knowledge between them. Then whole population reallocates in a group towards an minimum value. In particle swarm optimization, only g_{best} or l_{best} delivers exact information to others particles. This is a one-side information exchange process. Evolution optimization (EO) [44] only seeks the best solution to the problem. As compared to GA, all particles are connected to converge rapidly to best solution of the problem, though in local versions also in most cases.

PSO controlling parameters

In particle swarm optimization, It is necessary to adjust some parameters. Parameters list and their standard values are given here.

Number of particles range should be 10 to 50. Basically, for all the problems, 20 particles are so enough to book the best results.

Particles Dimensions: it is assumed through the problem of being optimized.

Range of particle: It is also assumed through problem to be optimum value, we should specify some ranges for different particle dimensions.

Condition of stop: Maximum iterations executed by PSO and error requirement will be minimum. Example, we can establish that the error requirement will be minimum is a zigzag pattern. Number of maximum iterations should be set to 2500. The stopping conditions will depend on the problem which is too optimized.

Local and global version: Here we know PSO's two versions that are global version as well as local version. The global version will be much faster but can converge to the local optimum values for most problems. The local version is slightly a bit slower, but it is not so easy to be captured for the local optimum values. Anyone may use the global version, so we get the fast results and use a local version to refine more to be search.

Global best model

In this approach, finding the best particle is possible between iteration; all best particles are modified before entering in the next level selection.

```
Swarm initialization
for loop i = 1 to
number of particles do
for j=1 to
number of dimensions do
initialize  $x_{ij}$  with a rnd ( $X_{min}$ ,  $X_{max}$ ) value
initialize  $v_{ij} = 0$ 
copy  $x_{ij}$  in  $p_{ij}$ 
end
end
search the best global leader and record its
position
swarm flight through the search space
do
for i=1 to
number of particles do
for j=1 to
number of dimensions do
update  $v_{ij}$  using  $p_{ij}$  and  $x_{ij}$ 
prevent an explosion of  $v_{ij}$ 
update  $x_{ij}$ 
if(loop_number<total_loop *probab_mut)
then mutate  $x_{ij}$ 
end for
evaluate fitness ( $x_i$ )
if fitness ( $p_i$ )< fitness ( $x_i$ )
then update  $p_i$ 
end for
search the best global leader and record its
position
while (loop_number<total_loop)
```

Fig 2: global – PSO algorithm Pseudo-code of global PSO model

Local best model

In this case, we always see the best particle, and hence these particles boost up the velocity of the particle within the neighborhood.

```

Swarm initialization
For i = 1 to
Number of particles do
For j = 1 to
Number of dimensions do
Initialize  $x_{ij}$  with a rnd ( $X_{min}$ ,  $X_{max}$ ) value
Initialize  $v_{ij}$  with zero value
Copy  $x_{ij}$  in  $p_{ij}$ 
End
End
Swarm flight through search space
do
For i = 1 to
number of particles do
For j = 1 to
number of dimensions do
Search in the k_neighborhood of particle  $x_i$ 
For j=1 to
Number of dimensions do
Update  $v_{ij}$  using  $p_{ij}$  and  $p_{ij}$ 
Prevent explosion of  $v_{ij}$ 
Update  $x_{ij}$ 
If (loop_number < otal_loop * probab_mut)

```

Fig 3: local – PSO algorithm Pseudo-code of local PSO model

Both models are proposed some results in this paper which follows Carlisle's discussion and suggestions [40].

Table 1 Year wise natural inspiration optimization algorithm

Year	Algorithms name	Inspiration
1975	Genetic Algorithms (GAs)[6]	Evolution
1983	Simulated annealing	Metallurgy
1995	Particle swarm optimization	Bird-flocking
2006	Ant colony optimization	Ant-colony
2006	Artificial bee colony [41]	Honey-bee
2010	Bat algorithm [42]	Echolocation

Test Functions

At first, we choose some multi-model functions to verify our concept. To find topology either the global best or the local best models, diversity is a key issue. This section compares different types of optimization algorithms using uni-model and non-uni-model test functions. The following functions are used here to test the optimization algorithms:-

1. **Rastrigin's function:** - It is a robust Spherical function and arranged as sinusoidal bumps, characterized by deep local minima. If $x^* = 0$, then global minimum is $f(x^*) = 0$.

$$F_1(x) = nA + x_i^2 - A \cos(wx_i) \quad (3)$$

Where $A = 10$ (It is a fix value), $w = 2 * 3.14$ and $F_1(x) = 0$ for $x = (0, 0, \dots, 0)$ is the global optimum.

Table 2 exhibits parameter settings for the functions. Here all the functions are tested using 40 – dimensional search spaces. Default parameters are used through-out to produce acceptable results of test functions.

2. **Spherical function:** - It is a very easy, uni-modal function. If $x^* = 0$, then global minimum is $f(x^*) = 0$.

$$F_2(x) = x_i^2 \quad (4)$$

Actually this function is a quadratic function.

3. **Ackley's function:** - It is a multi-model function as well as deep local minima. If $x^* = 0$, then global minimum is $f(x^*) = 0$. The variable used here are independent.

$$F_3(x) = -20 \exp(-0.2 \sqrt{1/n * x_i^2}) - \exp(1/n * \cos(wx_i)) + 20 + e \quad (5)$$

4. **Schwefel's function:** - If $x^* = 0$, then global minimum is $f(x^*) = 0$. The variable used here are independent.

$$F_4(x) = 418.98n + x_i \sin(\sqrt{x_i}) \quad (6)$$

Table 2 Function parameters

Function	N	Domain	Threshold
Ackley	30	030.00	005.00
Rastrigin	30	005.12	100.00
Spherical	30	100.00	000.01
Schwefel	30	100.00	000.01

Conclusions and future related work

PSOs are the very simple algorithms. It has a broad range of functions for optimizing problems. Our goal is to develop it. We have succeeded because of its easy and simple. These algorithms have some lines of code. The condition of the requirement is only some parameter of the problem. Moreover, it has to solve it using some specification. In conclusion, we propose some interesting research points for research purpose are as follows. There are various uncertainties in optimization problems, searching robust solutions for fitness evaluation

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