Neutrosophic Γ-Near Rings and Their Application to Cryptocurrency Risk Evaluation Using Multi-Criteria Decision-Making Techniques

S.Ragamayi[1], B. Krishnaveni[2], N. Konda reddy[3]

[1]Department of Engineering Mathematics, Koneru Lakshmaiah Education Foundation, Guntur 522502, Andhra Pradesh, India.

[2] Department of Mathematics, Aditya University, Surampalem 533437, Andhra Pradesh, India.

[3]Departmentof EngineeringMathematics,KoneruLakshmaiah EducationFoundation,Guntur522502,Andhra Pradesh,India.

Abstract: This paper introduces and investigates a novel algebraic structure known as the **neutrosophic Γ-near-ring**, which integrates neutrosophic set theory with Γ-near-ring frameworks to address operations under uncertainty, indeterminacy, and contradiction. Each element in the proposed structure is characterized by degrees of truth, indeterminacy, and falsity, thus enabling a more expressive and flexible modeling approach compared to traditional or fuzzy algebraic systems. Fundamental definitions are established, and key structural properties are explored.

It is verified that the **intersection** of two neutrosophic Γ-near-rings results in a neutrosophic Γ-near-ring, ensuring stability. Moreover, we demonstrate that the **union** of two neutrosophic Γ-near-rings also a neutrosophic Γ-near-ring, provided one is contained within the other. Additionally, a **one-one correspondence** is recognized amidst of the neutrosophic and crisp forms of a Γ-near-ring, ensuring logical consistency and reversibility.

The outcomes obtained here lay a theoretical foundation for further research and potential applications in various areas such as artificial intelligence, fuzzy logic, and algebraic modeling of uncertain systems.

Key words: Neutrosophic Γ-near-ring, fuzzy Γ-near-ring.

1. Introduction

Algebraic systems like near rings oblige as a central framework in abstract algebra owing to their relevance in logic circuits, automata design and coding theory [1], [2]. A significant generalization of this structure is the Γ-near-ring, wherein the operation "multiplication" is parameterized by a set Γ, contribution improved structural flexibility. This extension accommodates better sophisticated algebraic behavior and models systems governed by multi-dimensional relations [3].

The notion of the Γ-near-ring was presented by Bhavanari Satyanarayana in 1985 [2] as an extension of both near-rings and Γ-rings. This origination eased the study of ternary and vague algebraic structures. Building on this, Ragamayi et al. have contributed significantly by examining fuzzy, vague represents truth and false membership, and bipolar fuzzy ideals on a Γ-near-ring [4]– [10], which consider positive and negative aspects of membership, offering dual perspectives within the same algebraic framework [7]– [10].

In the realm of uncertainty modeling, fuzzy set theory—pioneered by Zadeh [3] in 1965—has laid the groundwork for representing partial membership in sets, challenging the binary nature of classical logic [11]. Atanassov later introduced intuitionistic fuzzy sets to incorporate both membership and non-membership degrees, enriching this framework [3]. However, to further address ambiguous, inconsistent, or incomplete data, Smarandache proposed neutrosophic sets, which represent every element with a triple (T, I, F), referring to the degrees of truth, indeterminacy, and falsity, respectively, where each component independently lies within [0, 1] [12], [13].

The neutrosophic approach has found application in various mathematical and decision-making contexts, offering nuanced models for systems where classical, fuzzy, or intuitionistic methods fall short [14], [15]. Recent developments extend neutrosophic theory to algebraic domains—such as neutrosophic quadruple Hv-rings and neutrosophic fuzzy algebras—highlighting its potential in algebraic structure modeling under uncertainty [16].

This paper presents a new integration of these ideas through the definition and study of neutrosophic Γ-near-rings, wherein elements are described not just by their structural placement but also by their associated neutrosophic values. We introduce foundational definitions, supply illustrative examples, and derive key properties. In particular: We prove that the intersection of neutrosophic Γ-near-rings yields a neutrosophic Γ-near-ring. We establish that the union of such rings also forms a neutrosophic Γ-near-ring, provided one is a subset of the other. A neutrosophic characteristic set for a Γ-near-ring is proposed. A one-to-one correspondence between the neutrosophic structure and its crisp counterpart is demonstrated.

These results not only contribute to theoretical understanding of neutrosophic algebra but also lay groundwork for its potential applications in modeling imprecise and multi-valued logical systems.

2. Preliminaries

We recall the basic notions and definitions regarding Γ-Near rings used in the paper.

Definition 2.1: A zero – symmetric GNR is a triple $(Ƚ, +, \Gamma)$, where
1. $(Ƚ, +)$ is a group
2. $(Ƚ, +, \delta)$ is a near ring where $\Gamma \neq \Phi$ with binary operators on $Ƚ$, $\forall \delta \in \Gamma$.
3. $f\ \delta(g\ \eta\ h) = (f\ \delta g)\ \eta h \forall f, g, h \epsilon Ƚ;\ \delta, \eta\ \epsilon\ \Gamma$.
4. $f\ \delta\ g = 0, \forall\ f\ \in\ Ƚ\ \delta \in \Gamma$.

Definition 2.2: A Fuzzy subset "ρ "of $Ƚ$ is a Fuzzy SGNR, if
1. $\rho\,(f - g) \geq \min\{\rho\,(f), \rho(g)\}$
2. $\rho\,(f\ \delta\ g) \geq \min\{\rho\,(f), \rho(g)\}, \forall \delta \in \Gamma; f, g\ \epsilon Ƚ$

Definition 2.3: A fuzzy subset $\rho : Ƚ \to [0, 1]$ is called a fuzzy left(resp. right) ideal of $Ƚ$ if for all f, g, a, b $\in Ƚ$; $\alpha \in \Gamma$
(1) $\rho\,(f - g) \geq \min\{\rho(f), \rho(g)\}$
(2) $\rho\,(g + f - g) \geq \rho(f)$ and
(3) $\rho\,(a\alpha\,(f + b) - a\alpha b) \geq \rho\,(f)$ (resp. $\rho\,(f\alpha a) \geq \rho\,(f)$.

Definition 2.4: Let $Ƚ$ be a space of points (objects), with a generic element. A neutrosophic set ϑ in $Ƚ$ is characterized by a truth-membership function $\vartheta_{tr}\,(l)$, an indeterminacy-membership function $\vartheta_{ind}(l)$ and a falsity-membership function $\vartheta_{fa}(l)$. Then, a neutrosophic set ϑ can be denoted by $\vartheta = \{< l, \vartheta_{tr}\,(l) + \vartheta_{fa}(l) + \vartheta_{ind}(l) >: l \in Ƚ\}$,
where $\vartheta_{tr}\,(l), \vartheta_{fa}(l), \vartheta_{ind}(l) \in [0, 1]\ \ \forall\ l$ in $Ƚ$ and $0 \leq \vartheta_{tr}\,(l) + \vartheta_{fa}(l) + \vartheta_{ind}(l) \leq 3$.

Definition 2.5: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ be a neutrosophic set then the complement of ϑ is defined as (ϑfa, ϑtr, $1 - \vartheta ind$).$\vartheta^c = (\vartheta^c_{tr}, \vartheta^c_{fa}, \vartheta^c_{ind}) = (\vartheta_{fa}, \vartheta_{tr}, 1 - \vartheta_{ind})$

Definition 2.6: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ and $\eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be two neutrosophic sets of a universe of discours e $Ƚ$ then the union of ϑ and η is denoted by $(\vartheta \cup \eta)$ which is defined as $(\vartheta \cup \eta) = ((\vartheta \cup \eta)_{tr}, (\vartheta \cup \eta)_{fa}, (\vartheta \cup \eta)_{ind})$, where
$(\vartheta \cup \eta)_{tr}\,(l) = \max\{\vartheta_{tr}\,(l), \eta_{tr}\,(l)\}$
$(\vartheta \cup \eta)_{fa}(l) = \min\{\vartheta_{fa}(l), \eta_{fa}(l)\}$
$(\vartheta \cup \eta)_{ind}(l) = \min\{\vartheta_{ind}(l), \eta_{ind}\,(l)\}\ \forall\ l$ in $Ƚ$.

Definition 2.7: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ be a neutrosophic set of a universe of discourse $Ƚ$. For α, β, γ $\in$ [0, 1] with $0 \leq \alpha + \beta + \gamma \leq 3$, the (α, β, γ)- cut or neutrosophic cut of ϑ is the crisp subset of $Ƚ$ is given by $\vartheta(\alpha, \beta, \gamma) = \{\, l \in Ƚ : \vartheta_{tr}\,(l) \geq \alpha,\ \vartheta_{fa}(l) \leq \beta,\ \vartheta_{ind}(l) \leq \gamma\}$.

Definition 2.8: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ and $\eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be two neutrosophic sets of a universe of discourse $\tilde{L}$ then $\vartheta \subseteq \eta$ iff $\vartheta_{tr}\,(l) \leq \eta_{tr}(l)$; $\vartheta_{fa}(l) \geq \eta_{fa}(l)$; $\vartheta_{ind}(l) \geq \eta_{ind}\,(l)$.

Definition 2.9: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ and $\eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be two neutrosophic sets of a universe of discourse $Ł$ then the intersection of ϑ and η is denoted by $(\vartheta \cap \eta)$ which is defined as

$(\vartheta \cap \eta) = ((\vartheta \cap \eta)_{tr}, (\vartheta \cap \eta)_{fa}, (\vartheta \cap \eta)_{ind})$, where
$(\vartheta \cap \eta)_{tr}(l) = \min\{\vartheta_{tr}(l), \eta_{tr}(l)\}$
$(\vartheta \cap \eta)_{fa}(l) = \max\{\vartheta_{fa}(l), \eta_{fa}(l)\}$
$(\vartheta \cap \eta)_{ind}(l) = \max\{\vartheta_{ind}(l), \eta_{ind}(l)\}\ \forall\ l$ in $Ł$.

3. On Neutrosophic Gamma Near rings

In this section, we formally introduce the concept of a **neutrosophic Γ-near-ring**, an algebraic structure that integrates the principles of neutrosophic set theory into the framework of Γ- near-rings. We establish several foundational results pertaining to this structure. Specifically:

- We **define** the neutrosophic Γ-near-ring, incorporating neutrosophic logic through triplet values of truth, indeterminacy, and falsity for each element.
- We **prove** that the **intersection** of two neutrosophic Γ-near-rings results in another neutrosophic Γ-near-ring, thereby demonstrating closure under intersection.
- We show that the **union** of two such structures also form a neutrosophic Γ-near-ring, provided one is a subset of the other..
- Furthermore, we establish a **one-to-one correspondence** between the neutrosophic representation of a Γ-near-ring and its associated crisp set. This correspondence ensures a reversible mapping between deterministic and neutrosophic formulations of the structure.

These theoretical developments provide a solid foundation for exploring deeper properties and potential applications of neutrosophic Γ-near-rings in environments where uncertainty and indeterminacy are inherent. Throughout this section $Ł$ stands for a Γ-Near ring unless otherwise mentioned.

Definition 3.1: A Neutrosophic set $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ is said to be a Neutrosophic Sub GNR , if

1. $\vartheta_{tr}(l - m) \geq \min\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}$
2. $\vartheta_{tr}(l\,\delta\,m) \geq \min\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}$
3. $\vartheta_{fa}(l - m) \leq \max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}$
4. $\vartheta_{fa}(l\,\delta\,m) \leq \max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}$
5. $\vartheta_{ind}(l - m) \leq \max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}$
6. $\vartheta_{ind}(l\,\delta\,m) \leq \max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}, \forall \delta \in \Gamma; l, m\ \epsilon Ł$

Definition 3.2: A Neutrosophic set $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ is mapping from $Ł \rightarrow [0, 1]$ is called a Neutrosophic left(resp. right) ideal of $Ł$ if for all $l, m, a, b\ \epsilon Ł,\ \delta \in \Gamma$;

1. $\vartheta_{tr}(l - m) \geq \min\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}$
2. $\vartheta_{tr}(m + l - m) \geq \vartheta_{tr}(l)$
3. $\vartheta_{tr}(a\delta(l + b) - a\,\delta\,b) \geq \vartheta_{tr}(l)$; (resp. $\vartheta_{tr}(l\,\delta\,a)) \geq \vartheta_{tr}(l)$)
4. $\vartheta_{fa}(l - m) \leq \max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}$
5. $\vartheta_{fa}(m + l - m) \leq \vartheta_{fa}(l)$
6. $\vartheta_{fa}(a\delta(l + b) - a\,\delta\,b) \leq \vartheta_{fa}(l)$; (resp. $\vartheta_{fa}(l\,\delta\,a)) \leq \vartheta_{fa}(l)$)
7. $\vartheta_{ind}(l - m) \leq \max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}$
8. $\vartheta_{ind}(m + l - m) \leq \vartheta_{ind}(l)$
9. $\vartheta_{ind}(a\delta(l + b) - a\,\delta\,b) \leq \vartheta_{ind}(l)$; (resp. $\vartheta_{ind}(l\,\delta\,a)) \leq \vartheta_{ind}(l)$)

Example 3.3: Let $Ł = \Gamma =$ "$\mathbb{R}$" (the set of Real numbers) which is clearly a GNR.

Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$, where $\vartheta_{tr} : Ł \rightarrow [0,1]$, $\vartheta_{fa} : Ł \rightarrow [0,1]$ *and* $\vartheta_{ind} : Ł \rightarrow [0,1]$

Defined by

$$\vartheta_{tr}(u) = \begin{cases} 0.41, & \text{if } u = 0 \\ 0.52, & \text{if } u > 0 \\ 0.73, & \text{if } u < 0 \end{cases} \quad \vartheta_{fa}(u) = \begin{cases} 0.32, & \text{if } u = 0 \\ 0.58, & \text{if } u > 0 \\ 0.31, & \text{if } u < 0 \end{cases} \quad \vartheta_{ind}(u) = \begin{cases} 0.11, & \text{if } u = 0 \\ 0.26, & \text{if } u > 0 \\ 0.71, & \text{if } u < 0 \end{cases}$$

Then ϑ is a neutrosophic GNR of $\boldsymbol{Ł}$.

Theorem 3.4: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ $and\ \eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be two neutrosophic ideals of a GNR of $Ł$ then the intersection of ϑ $and\ \eta$ is also a neutrosophic ideal of a GNR of $Ł$.

Proof: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ $and\ \eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be neutrosophic ideals of a GNR of $Ł$. Let $l, m, v \in Ł$; $\delta_1 \in \Gamma$.

$$\begin{aligned}(\vartheta \cap \eta)_{tr}\ (l-m) &= \min\{\vartheta_{tr}(l-m), \eta_{tr}(l-m)\}\\ &\geq \min\{\min\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}, \min\{\eta_{tr}(l), \eta_{tr}(m)\}\}\\ &= \min\{\min\{\vartheta_{tr}(l), \eta_{tr}(l)\}, \min\{\vartheta_{tr}(m), \eta_{tr}(m)\}\}\\ &= \min\{(\vartheta \cap \eta)_{tr}(l), (\vartheta \cap \eta)_{tr}(m)\}\end{aligned}$$

$$\begin{aligned}(\vartheta \cap \eta)_{fa}\ (l-m) &= \max\{\vartheta_{fa}(l-m), \eta_{fa}(l-m)\}\\ &\leq \max\{\max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}, \max\{\eta_{fa}(l), \eta_{fa}(m)\}\}\\ &= \max\{\max\{\vartheta_{fa}(l), \eta_{fa}(l)\}, \max\{\vartheta_{fa}(m), \eta_{fa}(m)\}\}\\ &= \max\{(\vartheta \cap \eta)_{fa}(l), (\vartheta \cap \eta)_{fa}(m)\}\end{aligned}$$

$$\begin{aligned}(\vartheta \cap \eta)_{ind}\ (l-m) &= \max\{\vartheta_{ind}(l-m), \eta_{ind}(l-m)\}\\ &\leq \max\{\max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}, \max\{\eta_{ind}(l), \eta_{ind}(m)\}\}\\ &= \max\{\max\{\vartheta_{ind}(l), \eta_{ind}(l)\}, \max\{\vartheta_{ind}(m), \eta_{ind}(m)\}\}\\ &= \max\{(\vartheta \cap \eta)_{ind}(l), (\vartheta \cap \eta)_{ind}(m)\}\end{aligned}$$

$$\begin{aligned}(\vartheta \cap \eta)_{tr}\ (m+l-m) &= \min\{\vartheta_{tr}(m+l-m), \eta_{tr}(m+l-m)\}\\ &\geq \min\{\vartheta_{tr}(l), \eta_{tr}(l)\}\\ &= (\vartheta \cap \eta)_{tr}(l)\end{aligned}$$

$$\begin{aligned}(\vartheta \cap \eta)_{fa}\ (m+l-m) &= \max\{\vartheta_{fa}(m+l-m), \eta_{fa}(m+l-m)\}\\ &\leq \max\{\vartheta_{fa}(l), \eta_{fa}(l)\}\\ &= (\vartheta \cap \eta)_{fa}(l)\end{aligned}$$

$$\begin{aligned}(\vartheta \cap \eta)_{ind}\ (m+l-m) &= \max\{\vartheta_{ind}(m+l-m), \eta_{ind}(m+l-m)\}\\ &\leq \max\{\vartheta_{ind}(l), \eta_{ind}(l)\}\\ &= (\vartheta \cap \eta)_{ind}(l)\end{aligned}$$

$$\begin{aligned}&(\vartheta \cap \eta)_{tr}\ (v\,\delta_1(m+l) - m\,\delta_1\,v)\\ &\quad= \min\{\vartheta_{tr}(v\,\delta_1(m+l) - m\,\delta_1\,v), \eta_{tr}(v\,\delta_1(m+l) - m\,\delta_1\,v)\}\\ &\quad\geq \min\{\vartheta_{tr}(l), \eta_{tr}(l)\}\\ &\quad= (\vartheta \cap \eta)_{tr}(l)\end{aligned}$$

$$(\vartheta \cap \eta)_{fa}\,(v\,\delta_1(m+l) - m\,\delta_1\,v)$$

$= \max \{\vartheta_{fa} (v\, \delta_1(m+l) - m\, \delta_1\, v), \eta_{fa}(v\, \delta_1(m+l) - m\, \delta_1\, v)\}$

$\leq \max\{ \vartheta_{fa}(l), \eta_{fa}(l)\}$

$= (\vartheta \cap \eta)_{fa}(l)$

$(\vartheta \cap \eta)_{ind}\ (v\, \delta_1(m+ l) - m\, \delta_1\, v)$

$= \max \{\vartheta_{ind} (v\, \delta_1(m+l) - m\, \delta_1\, v), \eta_{ind}(v\, \delta_1(m+l) - m\, \delta_1\, v)\}$

$\leq \max \{\vartheta_{ind}(l), \eta_{ind}(l)\}$

$= (\vartheta \cap \eta)_{ind}(l)$

$(\vartheta \cap \eta)_{tr}\ (l\, \delta_1\, v) = \min\{\vartheta_{tr}\ (l\, \delta_1\, v), \eta_{tr}(l\, \delta_1\, v)\}$

$\geq \min\{ \vartheta_{tr}(l), \eta_{tr}(l)\}$

$= (\vartheta \cap \eta)_{tr}(l)$

$(\vartheta \cap \eta)_{fa}\ (l\, \delta_1\, v) = \max \{\vartheta_{fa}\ (l\, \delta_1\, v), \eta_{fa}(l\, \delta_1\, v)\}$

$\leq \max\{ \vartheta_{fa}(l), \eta_{fa}(l)\}$

$= (\vartheta \cap \eta)_{fa}(l)$

$(\vartheta \cap \eta)_{ind}\ (l\, \delta_1\, v) = \max \{\vartheta_{ind}\ (l\, \delta_1\, v), \eta_{ind}(l\, \delta_1\, v)\}$

$\leq \max \{\vartheta_{ind}(l), \eta_{ind}(l)\}$

$= (\vartheta \cap \eta)_{ind}(l)$

Hence $(\vartheta \cap \eta)$ is a neutrosophic ideal of a GNR of $Ł$.

Corollary 3.5: The intersection of arbitrary family of neutrosophic ideals of a GNR $Ł$. is also a neutrosophic ideals of a GNR $Ł$.

Theorem 3.6: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ be a neutrosophic set. Then ϑ *is* a neutrosophic Γ-Nearring of $Ł$ if and only if the (α, β, γ)-cut $\vartheta_{(\alpha,\beta,\gamma)}$ is a sub Γ- Nearring of $Ł$.

Proof. Suppose $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ is a neutrosophic Γ-Nearring of $Ł$.

Let $l, m \in \vartheta_{(\alpha,\beta,\gamma)}$ *and* $\delta \in \Gamma$.

Then $\vartheta_{tr}(l), \vartheta_{tr}(m) \geq \alpha$ & $\vartheta_{fa}(l), \vartheta_{fa}(m)\} \leq \beta$ & $\vartheta_{ind}(l), \vartheta_{ind}(m)\} \leq \gamma$

Since ϑ is a neutrosophic Γ- Nearring of $Ł$, we have

1. $\vartheta_{tr}(l - m) \geq \min\{ \vartheta_{tr}(l), \vartheta_{tr}(m)\} \geq \alpha$
2. $\vartheta_{tr}(l\, \delta\, m) \geq \min\{ \vartheta_{tr}(l), \vartheta_{tr}(m)\} \geq \alpha$
3. $\vartheta_{fa}(l - m) \leq \max\{ \vartheta_{fa}(l), \vartheta_{fa}(m)\} \leq \beta$
4. $\vartheta_{fa}(l\, \delta\, m) \leq \max\{ \vartheta_{fa}(l), \vartheta_{fa}(m)\} \leq \beta$
5. $\vartheta_{ind}(l - m) \leq \max\{ \vartheta_{ind}(l), \vartheta_{ind}(m)\} \leq \gamma$
6. $\vartheta_{ind}(l\, \delta\, m) \leq \max\{ \vartheta_{ind}(l), \vartheta_{ind}(m)\} \leq \gamma$; $\forall \delta \in \Gamma; l, m \in Ł$.
   which implies $l - m$ and $l\, \delta\, m \in \vartheta_{(\alpha,\beta,\gamma)}$

Thus $\vartheta_{(\alpha,\beta,\gamma)}$ is a sub Γ-Nearring of $Ł$.

Conversely, suppose $\vartheta_{(\alpha,\beta,\gamma)}$ is a sub Γ-Nearring of $Ł$.

Let $l, m \in Ł$ and $\delta \in \Gamma$

Let $\vartheta_{tr}(l) \geq \alpha_{11}$; $\vartheta_{fa}(l)\} \leq \beta_{11}$; $\vartheta_{ind}(l)\} \leq \gamma_{11}$ and

$\vartheta_{tr}(m) \geq \alpha_{22}$; $\vartheta_{fa}(m)\} \leq \beta_{22}$; $\vartheta_{ind}(m)\} \leq \gamma_{22}$

Consider $\alpha = \min\{\alpha_{11}, \alpha_{22}\}$; $\beta = \max\{\beta_{11}, \beta_{22}\}$; $\gamma = \max\{\gamma_{11}, \gamma_{22}\}$.

Then $l, m \in \vartheta_{(\alpha,\beta,\gamma)}$ and $l - m$ and $l\, \delta\, m \in \vartheta_{(\alpha,\beta,\gamma)}$.

Hence

1. $\vartheta_{tr}(l - m) \geq \alpha = \min\{ \vartheta_{tr}(l), \vartheta_{tr}(m)\}$

2. $\vartheta_{tr}(l\,\delta\, m) \geq \alpha = \min\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}$
3. $\vartheta_{fa}(l-m) \leq \beta = \max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}$
4. $\vartheta_{fa}(l\,\delta\, m) \leq \beta = \max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}$
5. $\vartheta_{ind}(l-m) \leq \gamma = \max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}$
6. $\vartheta_{ind}(l\,\delta\, m) \leq \gamma = \max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}$,

Thus ϑ is a neutrosophic Γ- Nearring of $Ł$.

Theorem 3.7: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ *and* $\eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ of M be two neutrosophic ideals of a GNR, $Ł$ then the union of ϑ *and* η is also a neutrosophic ideals of a GNR, $Ł$, only if $\vartheta \subseteq \eta$ or $\eta \subseteq \vartheta$.

Proof: Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$ of $Ł$ *and* $\eta = (\eta_{tr}, \eta_{fa}, \eta_{ind})$ be neutrosophic sets.
Suppose $\vartheta \subseteq \eta$. Let $l, m \in Ł$ and $\delta 1 \in \Gamma$

$(\vartheta \cup \eta)_{tr}\,(l-m)) = \max\{\vartheta_{tr}\,(l-m), \eta_{tr}(l-m)\}$

$= \eta_{tr}(l-m)$

$\geq \min\{\eta_{tr}(l), \eta_{tr}(m)\}$

$= \min\{\max\{\vartheta_{tr}(l), \eta_{tr}(l)\}, \max\{\vartheta_{tr}(m), \eta_{tr}(m)\}\}$

$= \min\{\max\{\vartheta_{tr}(l), \vartheta_{tr}(m)\}, \max\{\vartheta_{tr}(l), \eta_{tr}(m)\}\}$

$= \min\{(\vartheta \cup \eta)_{tr}(l), (\vartheta \cup \eta)_{tr}(m)\}$

$(\vartheta \cup \eta)_{fa}\,(l-m)) = \min\{\vartheta_{fa}\,(l-m), \eta_{fa}(l-m)\}$

$= \eta_{fa}(l-m)$

$\leq \max\{\eta_{fa}(l), \eta_{fa}(m)\}$

$= \max\{\max\{\vartheta_{fa}(l), \eta_{fa}(l)\}, \max\{\vartheta_{fa}(m), \eta_{fa}(m)\}\}$

$= \max\{\max\{\vartheta_{fa}(l), \vartheta_{fa}(m)\}, \max\{\vartheta_{fa}(l), \eta_{fa}(m)\}\}$

$= max\{(\vartheta \cup \eta)_{fa}(l), (\vartheta \cup \eta)_{fa}(m)\}$

$(\vartheta \cup \eta)_{ind}\,(l-m)) = \min\{\vartheta_{ind}\,(l-m), \eta_{ind}(l-m)\}$

$= \eta_{ind}(l-m)$

$\leq \max\{\eta_{ind}(l), \eta_{ind}(m)\}$

$= \max\{\max\{\vartheta_{ind}(l), \eta_{ind}(l)\}, \max\{\vartheta_{ind}(m), \eta_{ind}(m)\}\}$

$= \max\{\max\{\vartheta_{ind}(l), \vartheta_{ind}(m)\}, \max\{\vartheta_{ind}(l), \eta_{ind}(m)\}\}$

$= max\{(\vartheta \cup \eta)_{ind}(l), (\vartheta \cup \eta)_{ind}(m)\}$

$(\vartheta \cup \eta)_{tr}\,(m+l-m)) = \max\{\vartheta_{tr}\,(m+l-m), \eta_{tr}(m+l-m)\}$

$= \eta_{tr}(m+l-m)$

$\geq \eta_{tr}(l)$

$= \max\{\vartheta_{tr}(l), \eta_{tr}(l)\}$

$= (\vartheta \cup \eta)_{tr}(l)$

$(\vartheta \cup \eta)_{fa}\,(m+l-m)) = \min\{\vartheta_{fa}\,(m+l-m), \eta_{fa}(m+l-m)\}$

$= \eta_{fa}(m+l-m)$

$\leq \eta_{fa}(l)$

$= \max\{\vartheta_{fa}(l), \eta_{fa}(l)\} = (\vartheta \cup \eta)_{fa}(l)$

$(\vartheta \cup \eta)_{ind}\ (m + l - m)) = \min\ \{\vartheta_{ind}\ (m + l - m), \eta_{ind}\ (m + l - m)\}$

$= \eta_{ind}\ (m + l - m)$

$\leq\ \ \eta_{ind}\ (l)$

$= \max\{\ \vartheta_{ind}(l), \eta_{ind}\ (l)\}$

$= (\vartheta \cup \eta)_{ind}(l)$

$(\vartheta \cup \eta)_{tr}\ \ (l\ \delta_1(m + v) - m\ \delta_1\ l)$

$= \max\ \{\vartheta_{tr}\ \ (ml\ \delta_1(m + v) - m\ \delta_1\ l), \eta_{tr}\ (l\ \delta_1(m+\ v) - m\ \delta_1\ l)\}$

$= \eta_{tr}\ (l\ \delta_1\ (m+\ v) - m\ \delta_1\ l)$

$\geq\ \ \eta_{tr}\ (v)$

$= \max\{\ \vartheta_{tr}\ (v), \eta_{tr}\ (v)\}$

$= (\vartheta \cup \eta)_{tr}\ (v)$

$(\vartheta \cup \eta)_{fa}\ (l\ \delta_1\ (m + v) - m\ \delta_1\ l)$

$= \min\ \{\vartheta_{fa}\ (ml\ \delta_1(m + v) - m\ \delta_1\ l), \eta_{fa}(l\ \delta_1\ (m+\ v) - m\ \delta_1\ l)\}$

$= \eta_{fa}\ (l\ \delta_1(m + v) - m\ \delta_1\ l)$

$\leq\ \ \eta_{fa}\ (v)$

$= \max\{\ \vartheta_{fa}(v), \eta_{fa}\ (v)\}$

$= (\vartheta \cup \eta)_{fa}(v)$

$(\vartheta \cup \eta)_{ind}\ (l\ \delta_1\ (m+\ v) - m\ \delta_1\ l)$

$= \min\ \{\vartheta_{ind}\ (ml\ \delta_1(m + v) - m\ \delta_1\ l), \eta_{ind}\ (l\ \delta_1(m + v) - m\ \delta_1\ l)\}$

$= \eta_{ind}\ (l\ \delta_1\ (m + v) - m\ \delta_1\ l)$

$\leq\ \ \eta_{ind}\ (v)$

$= \max\{\ \vartheta_{ind}(v), \eta_{ind}(v)\}$

$= (\vartheta \cup \eta)_{ind}(v)$

$(\vartheta \cup \eta)_{tr}\ \ (v\ \delta_1\ l)\ = \max\ \{\vartheta_{tr}\ \ (v\ \delta_1\ l), \eta_{tr}\ (v\ \delta_1\ l)\}$

$= \eta_{tr}\ (v\ \delta_1\ l)$

$\geq\ \ \eta_{tr}\ (v)$

$= \max\{\ \vartheta_{tr}\ (v), \eta_{tr}\ (v)\}$

$= (\vartheta \cup \eta)_{tr}\ (v)$

$(\vartheta \cup \eta)_{fa}\ \ (v\ \delta_1\ l)\ = \min\ \{\vartheta_{fa}\ (v\ \delta_1\ l), \eta_{fa}(v\ \delta_1\ l)\}$

$= \eta_{fa}\ (v\ \delta_1\ l)$

$\leq\ \eta_{fa}(v)$

$= \max\{\ \vartheta_{fa}(v), \eta_{fa}\ (v)\} = (\vartheta \cup \eta)_{fa}(v)$

$(\vartheta \cup \eta)_{ind}\ (v\ \delta_1\ l)\ = \min\ \{\vartheta_{ind}\ (v\ \delta_1\ l), \eta_{ind}\ (v\ \delta_1\ l)\}$

$= \eta_{ind}\ (v\ \delta_1\ l)$

$$\leq \eta_{ind}(v)$$

$$= \max\{\vartheta_{ind}(v), \eta_{ind}(v)\} = (\vartheta \cup \eta)_{ind}(v)$$

Therefore $(\vartheta \cup \eta)$ is also a neutrosophic Γ- Nearring of $\mathcal{L}$ if $\vartheta \subseteq \eta$ or $\eta \subseteq \vartheta$.

4. Application of Neutrosophic Gamma Near-Ring to assess Financial Risk based on Cryptocurrency Investment

Neutrosophic sets generalize fuzzy and intuitionistic fuzzy sets by handling **indeterminacy** explicitly. In algebraic systems like gamma near-rings, this enables modelling **operations with uncertain or partially defined mappings**. Useful in theoretical investigations where standard algebraic axioms may hold with **degrees of truth, falsity, and indeterminacy**. Gamma near-ring structures combined with neutrosophic logic help in **multi-criteria decision-making problems (MCDM) Under Uncertainty.** Ideal in scenarios with **conflicting, vague, or incomplete data** for example medical diagnosis, financial risk assessment.

Problem: Financial Risk Assessment Using Neutrosophic Gamma Near-Ring based on Cryptocurrency Investment.

Cryptocurrency souqs are vastly unpredictable, making it tough to assess financial hazards using traditional methods. By amalgamation neutrosophic logic (which handles uncertainty and incomplete information) with gamma near-ring structures, we catch a powerful tool for evaluating such complex situations. This method aids investors make better decisions by obviously representing uncertain or differing data in cryptocurrency investments.

Scenario: An stockholder is evaluating the risk of capitalizing in cryptocurrency, which is recognized for extreme volatility and regulatory uncertainty. Due to mixed market signals and rapidly fluctuating conditions, a neutrosophic gamma near-ring model is employed for risk analysis. The stockholder assesses each factor by means of neutrosophic values to interpretation for uncertain and differing market signals. Gamma near-ring operations permit amalgamation of expert views, fabricating a robust decision model.

Model Setup: Neutrosophic Γ-Near-Ring Structure

- Select $\mathcal{L}$ = {States of Possible Risk factors}
  = {Market-Volatility (MV), Regulatory-Pressure (RP), SocialMedia - Sentiment (SMS), Technology-Risk (TR), Liquidity (LQ) }
  = {MV, RP, SMS, TR , LQ}
- Γ = {States of Possible Expert Scenarios or Contextual }
  = {Crypto-Market-Analyst (CMA), Central-Bank-Report (CBR), Blockchain-Developer-Opinion (BDO), Market-data (MD)}
  ={CMA,CBR,BDO,MD}
- Operation: f: $\mathcal{L} \times \Gamma \to \mathcal{L}$ models how each risk factor behaves under varying expert viewpoints, forming a gamma near-ring structure. It means GNR operation Captures how various expert opinions influence the interpretation of each market factor.
- **Gamma-operation (γ)** is defined as mapping a Risk Factor (r ∈ $\mathcal{L}$) and a Scenario (γ ∈ Γ) to a new modified r resulting Risk Factor r′ ∈ $\mathcal{L}$ based on the scenario's influence.

Let $\vartheta = (\vartheta_{tr}, \vartheta_{fa}, \vartheta_{ind})$, where $\vartheta_{tr} : \mathcal{L} \to [0,1]$, $\vartheta_{fa} : \mathcal{L} \to [0,1]$ and $\vartheta_{ind} : \mathcal{L} \to [0,1]$ defined by

$$\vartheta_{tr}(r) = \begin{cases} 0.6, If\ r = MV \\ 0.4, if\ r = RP \\ 0.5, if\ r = SMS \\ 0.7, if\ r = Tr \\ 0.2\ if\ r = LQ \end{cases} \quad \vartheta_{fa}(r) = \begin{cases} 0.2, If\ r = MV \\ 0.2, if\ r = RP \\ 0.5, if\ r = SMS \\ 0.15, if\ r = Tr \\ 0.2\ if\ r = LQ \end{cases} \quad \vartheta_{ind} \begin{cases} 0.2, If\ r = MV \\ 0.4, if\ r = RP \\ 0.3, if\ r = SMS \\ 0.15, if\ r = Tr \\ 0.1\ if\ r = LQ \end{cases}$$

Γ = {Possible Expert Scenarios}

$\mathcal{L}$ =	**$\mathcal{L} \times \Gamma$**	**CMA**	**CBR**	**BDO**	**MD**	
	MV	MV	RP	*TR*	MV	
{Possible Risk factors}	**RP**	RP	RP	*TR*	RP	
	SMS	SMS	RP	*SMS*	MV	
	TR	TR	RP	*TR*	TR	
	LQ	MV	RP	LQ	LQ	

Then ϑ is a neutrosophic GNR of $\mathcal{L}$.

Neutrosophic Evaluation of Each Risk Factor (Cryptocurrency)

$\mathcal{L} \times \Gamma \rightarrow \mathcal{L}$	**CMA**	**CBR**	**BDO**	**MD**
MV	MV	RP	*TR*	MV
Neutrosophic Triplet (T, I, F),	(0.6, 0.2, 0.2)	(0.4, 0.4, 0.2)	*(0.5, 0.3, 0.2)*	(0.7, 0.15, 0.15)
Interpretation	Direct expert view; no change	Regulations influence volatility, increase indeterminacy	Tech flaws can cause sudden volatility	*Reliable statistics; higher truth, lower uncertainty*
RP	RP	RP	*TR*	RP
Neutrosophic Triplet (T, I, F),	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	*(0.5, 0.3, 0.2*	(0.45, 0.35, 0.2)
Interpretation	Expert adjusts based on current events	Source itself; unchanged	*Tech constraints indirectly affect regulation*	Slightly higher truth from public records
SMS	SMS	RP	*SMS*	MV
Neutrosophic Triplet (T, I, F),	(0.5, 0.3, 0.2)	(0.3, 0.5, 0.2)	(0.4, 0.4, 0.2)	(0.55, 0.25, 0.2)
Interpretation	Mixed sentiment: analyst confirms	Institutions are unsure how to react to hype	Developer skeptical of social buzz	Correlates with sudden price swings
TR	TR	RP	*TR*	TR
Neutrosophic Triplet (T, I, F),	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.65, 0.25, 0.1)
Interpretation	Analyst trusts tech less due to bugs	Regulation influenced by tech risks	Confident; evolving but secure	Real-world stability observed
LQ	MV	RP	LQ	LQ
Neutrosophic Triplet (T, I, F),	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.75, 0.15, 0.1)
Interpretation	Analyst links liquidity to market movement	Freeze risks in regulatory frameworks	Stable protocol supports access	Direct measure; no change

Aggregate Neutrosophic Investment Score for Cryptocurrency

Aggregate	Weighted Average Score
Truth (T)	0.59
Indeterminacy (I)	0.26
Falsity (F)	0.15

- **T (Truth)** indicates the degree of risk being confirmed by data or experts.
- **I (Indeterminacy)** represent uncertainty or lack of clarity in the observation.
- **F (Falsity)** shows contradiction or opposing evidence.

Investment Decision and Summary

- Moderate truth value suggests cautious optimism about cryptocurrency investment.
- High indeterminacy highlights major uncertainty, especially in regulations and social sentiment. Investment is suggested with diversification and risk hedging strategies.

Element	Interpretation
Gamma Near-Ring	Captures how various expert opinions influence the interpretation of each market factor
Neutrosophic Logic	Manages high uncertainty and inconsistent information in the crypto market
MCDM Outcome	Supports cautious investment with adaptive risk management

Conclusion:

The integration of neutrosophic sets with Gamma near-ring theory provides an effective framework for handling uncertainty in cryptocurrency risk analysis. It improves decision-making by considering truth, falsity, and indeterminacy. Overall, it offers a flexible approach for modern financial systems.

Future Scope:

The model can be enhanced using AI for real-time risk prediction. Efficient computational methods can improve its application to large datasets. It can also be extended to areas like healthcare and engineering.

References

[1] G. L. Booth, “A note on -near rings,” Stud. Sci. Math. Hungar., vol. 23, pp. 471–475, 1988.
[2] B. Satyanarayana, Contributions to Near-Rings Theory, Ph.D. dissertation, Nagarjuna University, 1984.
[3] L. A. Zadeh, “Fuzzy sets,” Inf. Control, vol. 8, pp. 338–353, 1965. doi: 10.1016/S0019-9958(65)90241-X.
[4] Y. B. Jun, M. Sapanic, and M. A. Öztürk, “Fuzzy ideals in gamma near-rings,” Turk. J. Math., vol. 22, pp. 449–459, 1998.
[5] S. Ragamayi, Y. Bhargavi, C. Krishnaveni, and P. Bindu, “Lattice-fuzzy prime-ideal of a gamma-near ring,” J. Crit. Rev., vol. 7, pp. 1–5, 2020. doi: 10.31838/jcr.07.13.01.
[6] S. Ragamayi and Y. Bhargavi, “A study of vague gamma-near rings,” Int. J. Sci. Technol. Res., vol. 9, pp. 3960–3963, 2020.
[7] S. Ragamayi and Y. Bhargavi, “Some results on homomorphism of vague ideal of a gamma-near ring,” Int. J. Sci. Technol. Res., vol. 9, pp. 3972–3975, 2020.
[8] V. P. V. Korada, S. Ragamayi, and A. Iampan, “Bipolar fuzzy filters of gamma-near rings,” Int. J. Anal. Appl., vol. 22, no. 2, 2024.
[9] V. P. V. Korada, S. Ragamayi, and A. Iampan, “A study on bipolar vague ideals of gamma-near rings,” Asia Pacific J. Math., vol. 11, 2024.
[10] V. P. V. Korada, S. Ragamayi, and A. Iampan, “Anti-homomorphisms in bipolar fuzzy ideals and bi-ideals of Γ-near rings,” Asia Pacific J. Math., vol. 11, no. 20, 2024.
[11] V. P. V. Korada, S. Ragamayi, and A. Iampan, “Bipolar fuzzy weak bi-ideals of gamma-near rings,” Asia Pacific J. Math., vol. 10, no. 46, 2023.
[12] F. Smarandache, “Neutrosophic set: A generalization of the intuitionistic fuzzy set,” Int. J. Pure Appl. Math., vol. 24, no. 3, 2004.
[13] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. 2005. [Online].
[14] J. Peng, J. Wang, J. Wang, H. Zhang, and X. Chen, “Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems,” Int. J. Syst. Sci., vol. 47, no. 10, pp. 2342–2358, 2016.
[15] R. Chatterjee, P. Majumdar, and S. K. Samanta, “Similarity measures in neutrosophic sets – I,” in Fuzzy Multi-Criteria Decision-Making Using Neutrosophic Sets, vol. 369, Springer, Cham, 2019, pp. 229–248. doi: 10.1007/978-3-030-00045- 5_11.
[16] K. P. Shanmugapriya and P. Hemavathi, “A novel concept of neutrosophic fuzzy sets in $\hat{Z}$-algebra,” in Recent Developments in Algebra and Analysis, Trends in Mathematics, Birkhäuser, Cham, 2024, pp. 47–56. doi: 10.1007/978-3-031-37538- 5_5.